

# Charged Cylindrical Collapse of Anisotropic Fluid

M. Sharif <sup>\*</sup> and Sundas Fatima <sup>†</sup>

Department of Mathematics, University of the Punjab,  
Quaid-e-Azam Campus, Lahore-54590, Pakistan.

## Abstract

Following the scheme developed by Misner and Sharp, we discuss the dynamics of gravitational collapse. For this purpose, an interior cylindrically symmetric spacetime is matched to an exterior charged static cylindrically symmetric spacetime using the Darmois matching conditions. Dynamical equations are obtained with matter dissipating in the form of shear viscosity. The effect of charge and dissipative quantities over the cylindrical collapse are studied. Finally, we show that homogeneity in energy density and conformal flatness of spacetime are necessary and sufficient for each other.

**Keywords:** Gravitational collapse; Dissipation; Junction conditions; Dynamical equations; Weyl tensor.

## 1 Introduction

A significant problem in gravitation theory and relativistic astrophysics is to understand the final fate of an endless gravitational collapse. A massive star undergoes a continual gravitational collapse at the end of its life cycle. This happens when a star has exhausted its nuclear fuel that provided a balance against the internal pull of gravity.

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<sup>\*</sup>msharif@math.pu.edu.pk

<sup>†</sup>sundas\_pu@yahoo.com

The importance of gravitational collapse in relativistic astrophysics was realized with the pioneer work of Oppenheimer and Snyder [1]. They used general relativity to study the dynamical collapse of a homogenous spherical dust cloud under its own gravity. Joshi and Singh [2] explored the spherically symmetric collapse of an inhomogeneous dust cloud. They found the end state of the gravitational collapse as a black hole or a naked singularity depending upon the initial density distribution and the radius of massive body. One may, however, consider dust as somewhat unrealistic form of matter, especially towards the end stages of a collapse, when pressures should be important. Keeping this fact in mind, the gravitational collapse of a perfect fluid and more general forms of matter have been studied. Misner and Sharp [3] discussed the gravitational collapse by taking the spherically symmetric ideal fluid in the interior and the Schwarzschild spacetime in the exterior of a star. They provided a full account of the dynamical equations governing the adiabatic relativistic collapse.

Darmois [4] presented junction conditions that joined two solutions of the Einstein field equations across the surfaces of discontinuity. Sharif and Ahmad [5] discussed junction conditions between static exterior and non-static interior spacetime in the presence of a positive cosmological constant. They also investigated the effect of a positive cosmological constant on spherically symmetric collapse with a perfect fluid. It was concluded that a positive cosmological constant slows down the rate of collapse. The same authors [6] also worked on cylindrical collapse of two perfect fluids using high speed approximation scheme and examined the effects of pressure on the high speed collapse for two possible cases. Kurita and Nakao [7] discussed the collapse of null dust in the cylindrically symmetric spacetime and found a naked singularity at the symmetric axis.

Gravitational collapse is a highly dissipative process [8]-[10] whose effects cannot be ignored in the study of collapse. Chan [11] studied a realistic model for a radiating star which undergoes dissipation in the form of radial heat flow and shear viscosity. He concluded that shear viscosity would increase anisotropy of pressure and also plays an important role in the study of gravitational collapse. The assumption of shear free motion of the fluid [12]-[14] is used to obtain exact solutions of the field equations but it is an unrealistic approach.

Herrera and Santos [9] studied dynamical description of gravitational collapse in view of Misner and Sharp's formulation. Matter under consideration was distributed with spherical symmetry and energy loss in the form of heat

flow and radiation. Herrera et al. [15] formulated the set of equations with regularity and matching conditions for the static cylindrically symmetric distribution of matter. They showed that any conformally flat cylindrically symmetric static source cannot be matched to the Levi-Civita spacetime by using Darmois junction conditions. One of the authors (LH) [16] discussed the inertia of heat and its role in the dynamics of dissipative collapse. Herrera et al. [17] also formulated the dynamical equations to include dissipation in the form of heat flow, radiation, shear and bulk viscosity and then coupled with causal transport equations. Recently, Sharif and Rehmat [18] extended this work for the plane symmetric gravitational collapse.

Some literature indicates keen interest for the inclusion of an electromagnetic field to discuss gravitational collapse. Bekenstein [19] generalized the Oppenheimer-Volkoff equations of hydrostatic equilibrium [20] and Misner-Sharp formulation for the dynamics of spherical gravitational collapse to the charged case. Nath et al. [21] explored gravitational collapse in the presence of electromagnetic field by using the junction conditions between quasi-spherical Szekeres spacetime in the interior and the charged Vaidya spacetime in the exterior region. They concluded that formation of a naked singularity was enhanced by an electromagnetic field. Sharif and Abbas [22] investigated the effect of an electromagnetic field on the spherically symmetric collapse with the perfect fluid in the presence of positive cosmological constant.

In a recent paper, Di Prisco et al. [23] derived dynamical equations for the spherically symmetric collapse by including an electromagnetic field. They concluded that Coulomb repulsion might prevent the gravitational collapse of the sphere. They also found the effect of charge on the relation between the Weyl tensor and the inhomogeneity of energy density. In this paper, we study the dynamics of a charged cylindrically symmetric spacetime to see the effect of charge on the rate of gravitational collapse.

The format of the paper is the following. In the next section, we describe the gravitational source and some physical quantities. The Einstein-Maxwell field equation are given in section **3** and junction conditions are derived in section **4**. We formulate the dynamical equations in section **5** and the relation between the Weyl tensor and the density homogeneity is given in section **6**. The last section sums up the main results of the paper.

## 2 Interior Matter Distribution and Some Physical Quantities

We consider a cylindrical surface with its motion described by a timelike three surface  $\Sigma$ , which divides  $4D$  spacetime into interior  $M^-$  and exterior  $M^+$  manifolds. We assume co-moving coordinates inside hypersurface  $\Sigma$ . The interior cylindrically symmetric metric is given by

$$ds_-^2 = -A^2 dt^2 + B^2 dr^2 + C^2(d\phi^2 + dz^2),$$

$$-\infty < t < +\infty, \quad 0 \leq r < +\infty, \quad 0 \leq \phi \leq 2\pi, \quad -\infty < z < +\infty \quad (1)$$

where  $\{\chi^{-\mu}\} \equiv \{t, r, \phi, z\}$  ( $\mu = 0, 1, 2, 3$ ) and  $A$ ,  $B$  and  $C$  are functions of  $t$  and  $r$ . Matter under consideration is anisotropic fluid which undergoes dissipation in the form of shear viscosity. The energy-momentum tensor for such a fluid is defined as

$$T_{\alpha\beta} = (\mu + P_{\perp})V_{\alpha}V_{\beta} + P_{\perp}g_{\alpha\beta} + (P_r - P_{\perp})\chi_{\alpha}\chi_{\beta} - 2\eta\sigma_{\alpha\beta}, \quad (2)$$

where  $\mu$ ,  $P_r$ ,  $P_{\perp}$ ,  $\eta$ ,  $V_{\alpha}$  and  $\chi_{\alpha}$  are the energy density, the radial pressure, the tangential pressure, the coefficient of shear viscosity, the four-velocity of the fluid and the unit four-vector along the radial direction respectively. These quantities satisfy

$$V^{\alpha}V_{\alpha} = -1, \quad \chi^{\alpha}\chi_{\alpha} = 1, \quad \chi^{\alpha}V_{\alpha} = 0. \quad (3)$$

The shear tensor  $\sigma_{ab}$  is defined by

$$\sigma_{\alpha\beta} = V_{(\alpha;\beta)} + a_{(\alpha}V_{\beta)} - \frac{1}{3}\Theta(g_{\alpha\beta} + V_{\alpha}V_{\beta}), \quad (4)$$

where the acceleration  $a_a$  and the expansion  $\Theta$  are given by

$$a_{\alpha} = V_{\alpha;\beta}V^{\beta}, \quad \Theta = V^{\alpha}_{;\alpha}. \quad (5)$$

Let it be mentioned here that the bulk viscosity does not appear explicitly as it has been absorbed in the form of radial and tangential pressures of the collapsing fluid. The four-velocity and the unit four-vector are given by

$$V^{\alpha} = A^{-1}\delta_0^{\alpha}, \quad \chi^{\alpha} = B^{-1}\delta_1^{\alpha}. \quad (6)$$

From Eqs.(4) and (6), the non-zero components of the shear tensor are

$$\sigma_{11} = \frac{2}{\sqrt{3}}B^2\sigma, \quad \sigma_{22} = \sigma_{33} = -\frac{1}{\sqrt{3}}C^2\sigma. \quad (7)$$

The shear scalar  $\sigma$  is defined by [24]

$$\sigma = \frac{1}{\sqrt{3}A} \left( \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right), \quad (8)$$

where dot refers to differentiation with respect to  $t$ . Thus we have

$$\sigma_{\alpha\beta}\sigma^{\alpha\beta} = 2\sigma^2. \quad (9)$$

Using Eqs.(5) and (6), it follows that

$$a_1 = \frac{A'}{A}, \quad \Theta = \frac{1}{A} \left( \frac{\dot{B}}{B} + 2\frac{\dot{C}}{C} \right), \quad (10)$$

where prime represents derivative with respect to  $r$ .

The C-energy for the cylindrical symmetric spacetime is defined by [25]

$$E = \frac{1}{8}(1 - l^{-2}\nabla^a r \nabla_a r), \quad (11)$$

where

$$\rho^2 = \xi_{(1)a}\xi_{(1)}^a, \quad l^2 = \xi_{(2)a}\xi_{(2)}^a, \quad r = \rho l.$$

Here  $\rho$  is the circumference radius,  $l$  is the specific length,  $r$  is the areal radius,  $\xi^a$  stands for Killing vectors of cylindrically symmetric spacetime and  $E$  represents the gravitational energy per specific length of the cylinder. Thus the specific energy of the cylinder with the contribution of electromagnetic field in the interior region can be written as [26]

$$E' = \frac{l}{8} + \frac{C}{2} \left( \frac{\dot{C}^2}{A^2} - \frac{C'^2}{B^2} \right) + \frac{s^2}{2C}. \quad (12)$$

### 3 The Field Equations

The Maxwell equations are given by

$$F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}, \quad (13)$$

$$F^{\alpha\beta}{}_{;\beta} = \mu_0 J^\alpha, \quad (14)$$

where  $F_{\alpha\beta}$  is the Maxwell field tensor,  $\phi_\alpha$  is the four-potential and  $J_\alpha$  is the four-current. We can write the electromagnetic energy-momentum tensor in the form

$$E_{\alpha\beta} = \frac{1}{4\pi} \left( F_\alpha^\gamma F_{\beta\gamma} - \frac{1}{4} F^{\gamma\delta} F_{\gamma\delta} g_{\alpha\beta} \right). \quad (15)$$

It is assumed that the charge is at rest with respect to the co-moving coordinates, thus the magnetic field is zero. Consequently, the four potential and the four current will become

$$\phi_\alpha = \Phi \delta_\alpha^0, \quad J^\alpha = \rho V^\alpha, \quad (16)$$

where  $\Phi = \Phi(t, r)$  is an arbitrary function and  $\rho = \rho(t, r)$  is the charge density. The charge conservation yields

$$s(r) = 2\pi \int_0^r \rho B C^2 dr, \quad (17)$$

where  $s(r)$  is the total electric charge of the interior. For the interior space-time, using Eqs.(6) and (16), the Maxwell equations take the following form

$$\Phi'' - \left( \frac{A'}{A} + \frac{B'}{B} - 2\frac{C'}{C} \right) \Phi' = \mu_0 \rho A B^2, \quad (18)$$

$$\dot{\Phi}' - \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} - 2\frac{\dot{C}}{C} \right) \Phi' = 0. \quad (19)$$

Solving these equations simultaneously, it follows that

$$\Phi' = \frac{\mu_0 s A B}{2\pi C^2}. \quad (20)$$

The Einstein field equations for the interior metric can be written

$$G_{\alpha\beta} = 8\pi(T_{\alpha\beta} + E_{\alpha\beta}). \quad (21)$$

Using Eqs.(1), (2), (6), (15) and (20), we can write the nonvanishing components as follows

$$\begin{aligned}
8\pi(T_{00} + E_{00}) &= 8\pi\mu A^2 + \frac{s^2\mu_0^2 A^2}{4\pi^2 C^4} \\
&= \frac{\dot{C}}{C} \left( 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \left( \frac{A}{B} \right)^2 \left( -2\frac{C''}{C} + \frac{C'}{C} \left( 2\frac{B'}{B} - \frac{C'}{C} \right) \right),
\end{aligned} \tag{22}$$

$$8\pi(T_{01} + E_{01}) = 0 = -2 \left( \frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{\dot{C}A'}{CA} \right), \tag{23}$$

$$\begin{aligned}
8\pi(T_{11} + E_{11}) &= 8\pi \left( P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) B^2 - \frac{s^2\mu_0^2 B^2}{4\pi^2 C^4} \\
&= - \left( \frac{B}{A} \right)^2 \left( 2\frac{\ddot{C}}{C} + \left( \frac{\dot{C}}{C} \right)^2 - 2\frac{\dot{A}\dot{C}}{AC} \right) + \left( \frac{C'}{C} \right)^2 + 2\frac{A'C'}{AC},
\end{aligned} \tag{24}$$

$$\begin{aligned}
8\pi(T_{22} + E_{22}) &= 8\pi \left( P_\perp + \frac{2}{\sqrt{3}}\eta\sigma \right) C^2 + \frac{s^2\mu_0^2}{4\pi^2 C^2} \\
&= - \left( \frac{C}{A} \right)^2 \left( \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}}{A} \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \frac{\dot{B}\dot{C}}{BC} \right) \\
&\quad + \left( \frac{C}{B} \right)^2 \left( \frac{A''}{A} + \frac{C''}{C} - \frac{A'}{A} \left( \frac{B'}{B} - \frac{C'}{C} \right) - \frac{B'C'}{BC} \right).
\end{aligned} \tag{25}$$

Equation (23) can be re-written in the following form

$$\frac{1}{3}(\Theta - \sqrt{3}\sigma)' - \sqrt{3}\sigma\frac{C'}{C} = 0. \tag{26}$$

## 4 Junction Conditions

In this section, we formulate the junction conditions for the interior and exterior manifolds. The interior manifold is given by Eq.(1) and the exterior manifold is the charged static cylindrically symmetric spacetime given by [27]

$$ds_+^2 = -NdT^2 + \frac{1}{N}dR^2 + R^2(d\phi^2 + dz^2), \tag{27}$$

where

$$N(R) = \left( \frac{Q^2}{R^2} - 2\frac{M}{R} \right)$$

and  $\chi^{+\mu} \equiv \{T, R, \phi, z\}$ . We can write the metric for the hypersurface  $\Sigma$  in the following form

$$(ds^2)_\Sigma = -d\tau^2 + A^2(\tau)(d\phi^2 + dz^2), \quad (28)$$

where  $\xi^i \equiv (\tau, \phi, z)$  ( $i = 0, 2, 3$ ) represent the intrinsic coordinates of  $\Sigma$ .

The Darmois junction conditions [4] can be stated as follows:

- The continuity of the first fundamental form. This implies the continuity of the metrics over the hypersurface

$$(ds^2)_\Sigma = (ds_-^2)_\Sigma = (ds_+^2)_\Sigma. \quad (29)$$

- The continuity of the second fundamental form. This gives the continuity of the extrinsic curvature  $K_{ij}$  over the hypersurface

$$[K_{ij}] = K_{ij}^+ - K_{ij}^- = 0. \quad (30)$$

$K_{ij}^\pm$  is the extrinsic curvature given by

$$K_{ij}^\pm = -n_\sigma^\pm \left( \frac{\partial^2 \chi_\pm^\sigma}{\partial \xi^i \partial \xi^j} + \Gamma_{\mu\nu}^\sigma \frac{\partial \chi_\pm^\mu}{\partial \xi^i} \frac{\partial \chi_\pm^\nu}{\partial \xi^j} \right), \quad (\sigma, \mu, \nu = 0, 1, 2, 3). \quad (31)$$

where  $n_\sigma^\pm$  are the components of outward unit normal to the hypersurface in the coordinates  $\chi^{\pm\mu}$ .

We can write the equations of the hypersurface as follows:

$$f_-(t, r) = r - r_\Sigma = 0, \quad (32)$$

$$f_+(T, R) = R - R_\Sigma(T) = 0, \quad (33)$$

where  $r_\Sigma$  is a constant. Using Eqs.(32) and (33) in Eqs.(1) and (27), we have the interior and exterior spacetimes on  $\Sigma$  respectively

$$(ds_-^2)_\Sigma = -A^2(t, r_\Sigma)dt^2 + C^2(t, r_\Sigma)(d\phi^2 + dz^2). \quad (34)$$

$$(ds_+^2)_\Sigma = - \left[ N(R_\Sigma) - (N(R_\Sigma))^{-1} \left( \frac{dR_\Sigma}{dT} \right)^2 \right] dT^2 + R_\Sigma^2(d\phi^2 + dz^2). \quad (35)$$



The continuity of the first fundamental form gives

$$R_\Sigma^2 = C(t, r_\Sigma), \quad (36)$$

$$\frac{dt}{d\tau} = \frac{1}{A}, \quad (37)$$

$$\frac{dT}{d\tau} = \left[ N(R_\Sigma) - N(R_\Sigma)^{-1} \left( \frac{dR_\Sigma}{dT} \right)^2 \right]^{-\frac{1}{2}}. \quad (38)$$

Now we consider the second fundamental form over  $\Sigma$ . For this purpose, we need the outward unit normals to  $\Sigma$  using Eqs.(32) and (33),

$$n_\mu^- = B(0, 1, 0, 0), \quad (39)$$

$$n_\mu^+ = \left[ N(R) - (N(R))^{-1} \left( \frac{dR}{dT} \right)^2 \right]^{-\frac{1}{2}} \left( -\frac{dR}{dT}, 1, 0, 0 \right). \quad (40)$$

The surviving components of the extrinsic curvature  $K_{ij}^\pm$  can be given as follows

$$K_{00}^- = - \left( \frac{A'}{AB} \right)_\Sigma, \quad (41)$$

$$K_{00}^+ = \left[ \frac{d^2 T}{d\tau^2} \frac{dR}{d\tau} - \frac{d^2 R}{d\tau^2} \frac{dT}{d\tau} - \frac{N}{2} \frac{dN}{dR} \left( \frac{dT}{d\tau} \right)^3 + \frac{3}{2N} \frac{dN}{dR} \left( \frac{dR}{d\tau} \right)^2 \frac{dT}{d\tau} \right]_\Sigma \quad (42)$$

$$K_{22}^- = K_{33}^- = \left( \frac{CC'}{B} \right)_\Sigma, \quad (43)$$

$$K_{22}^+ = K_{33}^+ = \left[ NR \frac{dT}{d\tau} \right]_\Sigma. \quad (44)$$

The continuity of the second fundamental form, using Eq.(30), yields

$$\left[ \frac{d^2 T}{d\tau^2} \frac{dR}{d\tau} - \frac{d^2 R}{d\tau^2} \frac{dT}{d\tau} - \frac{N}{2} \frac{dN}{dR} \left( \frac{dT}{d\tau} \right)^3 + \frac{3}{2N} \frac{dN}{dR} \left( \frac{dR}{d\tau} \right)^2 \frac{dT}{d\tau} \right]_\Sigma = - \left( \frac{A'}{AB} \right)_\Sigma, \quad (45)$$

$$\left[ NR \frac{dT}{d\tau} \right]_\Sigma = \left( \frac{CC'}{B} \right)_\Sigma. \quad (46)$$

Making use of Eq.(38), it follows

$$\frac{dT}{d\tau} = \frac{1}{N} \sqrt{N + \left(\frac{dR}{d\tau}\right)^2}. \quad (47)$$

Substituting Eq.(47) in Eq.(46), we obtain

$$M = \frac{C}{2} \left( \frac{\dot{C}^2}{A^2} - \frac{C'^2}{B^2} \right) + \frac{Q^2}{2C}. \quad (48)$$

Thus, if the interior and the exterior charges are equal over the hypersurface, i.e.,  $Q = s$ , then we have

$$E' - \frac{l}{8} \stackrel{\Sigma}{=} M. \quad (49)$$

The difference between these two masses is equal to  $\frac{l}{8}$ , which is due to the least unsatisfactory definition of Thorne C-energy. Differentiating Eq.(47) with respect to  $\tau$ , then inserting this value in Eq.(45) and making use of Eq.(46), we can write Eq.(45) as

$$\frac{\dot{C}'}{C} - \frac{\dot{B}C'}{BC} - \frac{A'\dot{C}}{AC} = 0 \quad (50)$$

This equation identically satisfies Eq.(23). For the smooth matching of the interior and exterior metrics on hypersurface, Eqs.(36)-(38), (48) and (50) are the necessary and sufficient conditions.

## 5 Dynamical Equations

The conservation of energy-momentum,  $(T^{\alpha\beta} + E^{\alpha\beta})_{;\beta} = 0$ , implies that

$$\begin{aligned} (T^{\alpha\beta} + E^{\alpha\beta})_{;\beta} V_\alpha &= -\frac{\dot{\mu}}{A} - \frac{\dot{B}}{AB} \left( \mu + P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) \\ &\quad - \frac{2\dot{C}}{AC} \left( \mu + P_\perp + \frac{2}{\sqrt{3}}\eta\sigma \right) = 0 \end{aligned} \quad (51)$$

and

$$\begin{aligned} (T^{\alpha\beta} + E^{\alpha\beta})_{;\beta} \chi_a &= \frac{1}{B} \left( P_r - \frac{4}{\sqrt{3}}\eta\sigma \right)' + \frac{A'}{AB} \left( \mu + P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) \\ &\quad + \frac{2C'}{BC} \left( P_r - P_\perp - 2\sqrt{3}\eta\sigma \right) - \frac{\mu_0^2 ss'}{16\pi^3 BC^4} = 0. \end{aligned} \quad (52)$$

In view of Misner and Sharp's formalism, we discuss the dynamics of a collapsing system. We introduce the proper time derivative as

$$D_T = \frac{1}{A} \frac{\partial}{\partial t}. \quad (53)$$

The proper radial derivative  $D_R$  constructed from the circumference radius of a cylinder inside  $\Sigma$  is

$$D_R = \frac{1}{R'} \frac{\partial}{\partial r}, \quad (54)$$

where

$$R = C. \quad (55)$$

The fluid velocity in the case of collapse can be defined as

$$U = D_T(R) = D_T(C) < 0, \quad (56)$$

which must be negative in the process of collapse. Using Eq.(56), we can re-write Eq.(12) as

$$\tilde{E} = \frac{C'}{B} = \left[ U^2 + \frac{s^2}{C^2} - \frac{2}{C} \left( E' - \frac{1}{8} \right) \right]^{1/2}. \quad (57)$$

From Eqs.(54) and (55), Eq.(26) can be written as

$$BE \left[ \frac{1}{3} D_R(\Theta - \sqrt{3}\sigma) - \sqrt{3} \frac{\sigma}{R} \right] = 0.$$

In non-dissipative shear free case, i.e.,  $\eta = \sigma = 0$ , this equation takes the form

$$D_R \left( \frac{U}{R} \right) = 0.$$

This means that  $U \sim R$  which describes the homologous collapse. In view of Eqs.(12), (23), (24) and (53), the rate of variation of the C-energy turns out to be

$$D_T E' = -4\pi R^2 \left( P_r - \frac{4}{\sqrt{3}} \eta \sigma - \frac{1}{32\pi R^2} \right) U + \frac{s^2 U}{R^2} \left( \frac{\mu_0^2}{8\pi^2} - \frac{1}{2} \right). \quad (58)$$

The first term on the right-hand side of Eq.(58) in the case of collapse ( $U < 0$ ) will increase the energy of the cylinder if

$$P_r - \frac{4}{\sqrt{3}}\eta\sigma > \frac{1}{32\pi R^2}, \quad (59)$$

i.e., the effective radial pressure is greater than the particular value. This increase of C-energy is due to the work being done by the effective radial pressure. The second term in the round brackets describe energy leaving the system due to the Coulomb repulsive force. Similarly, using Eqs.(12), (22), (23) and (54), we obtain

$$D_R E' = 4\pi\mu R^2 + \frac{1}{8} + \frac{s}{R} D_R s + \frac{s^2}{R^2} \left( \frac{\mu_0^2}{8\pi^2} - \frac{1}{2} \right). \quad (60)$$

This equation indicates variation of the total energy between adjoining cylindrical surfaces inside the fluid distribution. The first term on right hand side gives the contribution of the energy density of the fluid element and the remaining terms are a constant and the electromagnetic contribution respectively. Integration of Eq.(60) leads to

$$E' = \int_0^R 4\pi\mu R^2 dR + \frac{R}{8} + \frac{s^2}{2R} + \frac{\mu_0^2}{8\pi^2} \int_0^R \frac{s^2}{R^2} dR. \quad (61)$$

Using Eqs.(12), (24), (56) and (57), we can obtain the acceleration  $D_T U$  of a collapsing matter inside  $\Sigma$  as follows:

$$D_T U = -\frac{1}{R^2} \left( E' - \frac{l}{8} \right) - 4\pi R \left( P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) + \frac{\tilde{E}A'}{AB} + \frac{s^2}{R^3} \left( \frac{\mu_0^2}{8\pi^2} + \frac{1}{2} \right). \quad (62)$$

Inserting the value of  $\frac{A'}{A}$  from Eq.(62) into Eq.(52), it follows that

$$\begin{aligned} & \left( \mu + P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) D_T U = - \left( \mu + P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) \\ & \times \left[ \frac{1}{R^2} \left( E' - \frac{l}{8} \right) + 4\pi \left( P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) R - \frac{s^2}{R^3} \left( \frac{\mu_0^2}{8\pi^2} + \frac{1}{2} \right) \right] \\ & - \tilde{E}^2 \left[ D_R \left( P_r - \frac{4}{\sqrt{3}}\eta\sigma \right) + 2 \left( P_r - P_\perp - 2\sqrt{3}\eta\sigma \right) \frac{1}{R} - \frac{\mu_0^2 s}{16\pi^3 R^4} D_R s \right]. \end{aligned} \quad (63)$$

This equation has the “Newtonian” form, i.e.,

$$\text{Mass density} \times \text{Acceleration} = \text{Force}.$$

Now we analyse the terms appearing in this equation as follows: The term in the round brackets on the left hand side represents inertial mass density. This gives the effect of the dissipative terms but there is no contribution of the electric charge. The remaining term on the left hand side is acceleration. There are two main terms on the right hand side. The first term represents the gravitational force. The factor within the round brackets is the same as on the left. It represents passive gravitational mass density by equivalence principle. The factor within the first square brackets shows how specific length, dissipation, and the electric charge affect the active gravitational mass term. Making use of Eq.(61) in Eq.(63), it turns out

$$\int_0^R \frac{s^2}{R^2} dR > \frac{s^2}{R},$$

which on differentiation yields

$$\frac{s}{R} > D_R s. \quad (64)$$

Thus the charge will increase the active gravitational mass only if this condition is satisfied. This increase of active gravitational mass causes the rapid collapse.

The second term in the second square brackets constitute hydrodynamical forces. It consists of further three terms. The first contribution simply represents the gradient of the total effective radial pressure (including the influence of shear viscosity on  $P_r$ ) which is always negative and is directed outward, this would prevent the gravitational collapse. The second term exhibits the effect of the local anisotropy of pressure with shear viscosity. If anisotropic pressure is positive then it increases the rate of collapse otherwise it decreases. In the last term, we have the Coulomb repulsion which may prevent the gravitational collapse of the cylinder.

For hydrostatic equilibrium, i.e.,  $U = 0$ ,  $\eta = 0$ , Eq.(63) turns out to be

$$\begin{aligned} D_R P_r &= \frac{\mu_0^2 s}{16\pi^3 R^4} D_R s - 2(P_r - P_\perp) \\ &- \frac{C'^2}{B^2} (\mu + P_r) \left[ \frac{1}{R^2} \left( E' - \frac{l}{8} \right) + 4\pi P_r R - \frac{s^2}{R^3} \left( \frac{\mu_0^2}{8\pi^2} + \frac{1}{2} \right) \right] \end{aligned} \quad (65)$$

We can also write Eq.(52) for the hydrostatic equilibrium  $U = 0$ ,  $\eta = 0$  as follows:

$$P'_r + \frac{A'}{A}(\mu + P_r) + \frac{2C'}{C}(P_r - P_\perp) - \frac{\mu_0^2 s s'}{16\pi^3 C^4} = 0. \quad (66)$$

This corresponds to the hydrostatic equilibrium for the spherically symmetric case and also gives the generalization of the Tolman-Oppenheimer-Volkoff equation for anisotropic charged fluid [28].

The static fluid leads to charged dust by taking  $P_r = 0 = P_\perp$ , thus, it follows from Eqs.(66) and (17) that

$$\mu \frac{A'}{A} - \frac{\mu_0^2 s \rho B}{8\pi^2 C^2} = 0. \quad (67)$$

Here  $B$  and  $C$  are function of  $r$  and hence  $B = C$  for a suitable transformation of  $r$ . Eliminating  $s$  from the field equations (24) and (25), we get

$$B = C, \quad AB = 1, \quad s^2 = \frac{4\pi^2}{\mu_0^2} B'^2. \quad (68)$$

Inserting these values in in Eq.(67), we obtain

$$\mu^2 = \kappa \rho^2, \quad \kappa = \frac{\mu_0^2}{16\pi^2}. \quad (69)$$

We would like to mention here that such type of result was also found by Bonnor [29] for arbitrary spacetime.

## 6 The Weyl Tensor

Here we shall explore the relation between the Weyl tensor and density inhomogeneity. For this purpose, we define the Weyl scalar  $\mathcal{C}^2$  in terms of the Kretchman scalar  $\mathcal{R}$ , the Ricci tensor  $R_{\alpha\beta}$  and the curvature scalar  $R$ , i.e.,

$$\mathcal{C}^2 = \mathcal{R} - 2R^{\alpha\beta}R_{\alpha\beta} + \frac{1}{3}R^2. \quad (70)$$

Inserting the value of  $\mathcal{R}$  from Eq.(88) in the appendix and making use of Eqs.(22)-(25) in Eq.(70), it follows that

$$\varepsilon = E' - \frac{l}{8} - \frac{4\pi}{3}R^3 \left( \mu - P_r + P_\perp + 2\sqrt{3}\eta\sigma \right) - \frac{s^2}{R} \left( \frac{\mu_0^2}{8\pi^2} + \frac{1}{2} \right), \quad (71)$$

where  $\varepsilon$  is defined as

$$\varepsilon = \frac{\mathcal{C}}{48^{\frac{1}{2}}} R^3. \quad (72)$$

Applying the definitions of  $D_T$  and  $D_R$  from Eqs.(58) and (60) respectively to (71), we obtain

$$\begin{aligned} D_T \varepsilon &= -4\pi \left[ \frac{1}{3} R^3 D_T \left( \mu - P_r + P_\perp + 2\sqrt{3}\eta\sigma \right) \right. \\ &\quad \left. + \left( \mu + P_\perp + \frac{2}{\sqrt{3}}\eta\sigma \right) U R^2 \right] + \frac{\mu_0^2 s^2 U}{4\pi^2 R^2}. \end{aligned} \quad (73)$$

and

$$\begin{aligned} D_R \varepsilon &= 4\pi \left[ -\frac{1}{3} R^3 D_R \left( \mu - P_r + P_\perp + 2\sqrt{3}\eta\sigma \right) \right. \\ &\quad \left. + \left( P_r - P_\perp - 2\sqrt{3}\eta\sigma \right) R^2 \right] + \frac{\mu_0^2}{4\pi^2} \left[ -\frac{s D_R s}{R} + \left( \frac{s}{R} \right)^2 \right]. \end{aligned} \quad (74)$$

This shows that production of density inhomogeneity is directly linked to dissipative variables and charge. For the case of zero charge and dissipation, we have

$$D_R \varepsilon + \frac{4\pi}{3} R^3 D_R \mu = 0. \quad (75)$$

This implies that if  $D_R \mu = 0$  then  $\mathcal{C} = 0$  (using the regular axis condition). Conversely, the conformally flat condition implies homogeneity in the energy density. This result has already been verified for spherically symmetric gravitational collapse [23].

## 7 Summary and Conclusion

To investigate how the system gradually changes with time, we have formulated a dynamical description of the cylindrically symmetric spacetime using Misner and Sharp's approach. Dissipative effects and anisotropic pressure have been taken into account. The junction conditions between cylindrically symmetric in the interior and charged static cylindrically symmetric spacetime in the exterior provides the gravitational mass which causes gravity in the exterior region.

We have found the behavior of charge and pressure through the dynamical equations. It turns out that electric charge (unlike pressure) does not always produce a regeneration effect, i.e., the pressure trying to keep the star in equilibrium through the pressure gradients, at the same time contributes to the active gravitational mass. Thus it increases the gravitational attraction and hence it promotes stellar collapse at the same time. This is due to the inequality (63) which implies that if

$$\frac{s}{R} > D_R s,$$

then it decreases the gravitational mass and also due to the Coulomb force that always opposes the gravitational force. We would like to mention here that our results indicate similarity with those found for the spherically symmetric spacetime [23].

We have also established an expression indicating the relevance of the electric charge with the Weyl tensor and density inhomogeneity. Using the regular axis condition, it has been shown that homogeneity in energy density and conformal flatness of spacetime are necessary and sufficient for each other. We would like to mention here that the Weyl tensor contains tidal forces that make the fluid more inhomogeneous in the process of evolution. It would be interesting to include also a heat flux and examine the corresponding transport equations. Also, one would be interested to extend these results for charged plane symmetric spacetime [30].

## Appendix

The interior metric has the following nonvanishing components of the Riemann tensor

$$R_{0101} = AA'' - B\ddot{B} - \frac{A}{B}A'B' + \frac{B}{A}\dot{A}\dot{B}, \quad (76)$$

$$R_{0202} = \frac{C}{AB^2} \left( -\ddot{C}AB^2 + \dot{A}\dot{C}B^2 + A'C'A^2 \right), \quad (77)$$

$$R_{0212} = \frac{C}{AB} \left( -\dot{C}'AB + \dot{C}A'B + \dot{B}C'A \right), \quad (78)$$

$$R_{1212} = \frac{C}{A^2B} \left( -C''A^2B + \dot{C}\dot{B}B^2 + B'C'A^2 \right), \quad (79)$$

$$R_{2323} = \frac{C^2}{(AB)^2} (\dot{C}^2 B^2 - C'^2 A^2), \quad (80)$$



and

$$R_{0202} = R_{0303}, \quad R_{0212} = R_{0313}, \quad R_{1212} = R_{1313}. \quad (81)$$

Thus it has five independent components. The Kretschman scalar  $\mathcal{R} = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$  becomes

$$\begin{aligned} \mathcal{R} = 4[ & \frac{1}{(AB)^4}(R_{0101})^2 + \frac{2}{(AC)^4}(R_{0202})^2 - \frac{4}{(AB)^2C^4}(R_{0212})^2 \\ & + \frac{2}{(BC)^4}(R_{1212})^2 + \frac{1}{C^8}(R_{2323})^2]. \end{aligned} \quad (82)$$

The components of the Riemann tensor in terms of the Einstein tensor and the C-energy function can be written as

$$R_{0101} = (AB)^2 \left[ \frac{1}{2A^2}G_{00} - \frac{1}{2B^2}G_{11} + \frac{G_{22}}{C^2} - \frac{2}{C^3} \left( E' - \frac{l}{8} - \frac{s^2}{2C} \right) \right] \quad (83)$$

$$R_{0202} = (AC)^2 \left[ \frac{G_{11}}{2B^2} + \frac{1}{C^3} \left( E' - \frac{l}{8} - \frac{s^2}{2C} \right) \right], \quad (84)$$

$$R_{0212} = \frac{C^2}{2}G_{01}, \quad (85)$$

$$R_{1212} = (BC)^2 \left[ \frac{G_{00}}{2A^2} - \frac{1}{C^3} \left( E' - \frac{l}{8} - \frac{s^2}{2C} \right) \right], \quad (86)$$

$$R_{2323} = 2C \left( E' - \frac{l}{8} - \frac{s^2}{2C} \right). \quad (87)$$

Inserting Eqs.(83)-(87) into Eq.(82), we get

$$\begin{aligned} \mathcal{R} = & \frac{48}{C^6} \left( E' - \frac{l}{8} - \frac{s^2}{2C} \right)^2 - \frac{16}{C^3} \left( E' - \frac{l}{8} - \frac{s^2}{2C} \right) \left[ \frac{G_{00}}{A^2} - \frac{G_{11}}{B^2} + \frac{G_{22}}{C^2} \right] \\ & - 4 \left( \frac{G_{01}}{AB} \right)^2 + 3 \left[ \left( \frac{G_{00}}{A^2} \right)^2 + \left( \frac{G_{11}}{B^2} \right)^2 \right] + 4 \left( \frac{G_{22}}{C^2} \right)^2 \\ & - 2 \frac{G_{00}G_{11}}{(AB)^2} + 4 \left( \frac{G_{00}}{A^2} - \frac{G_{11}}{B^2} \right) \frac{G_{22}}{C^2}. \end{aligned} \quad (88)$$

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